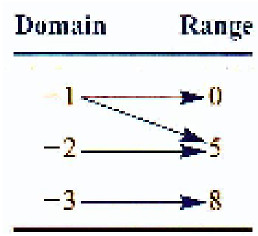
1) Which of the following relations represent functions?

I. $(x-3)^2 + y^2 = 9$

=> **Not a function**. When $x = 3$ (for example) you have $y^2 = 9$ … which yields two solutions: -3, 3.

II. $\{(0,4),(6,4),(10,4)\}$

=> **A function**. The *x* values are unique, and so no *x* points to two different *y* s.

III. 

Not a function - -1 in the domain yields both 0 and 5 in the range. Multiple values for a single *x* signify **relation** and not function.

So – the **correct answer is A**

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2) Determine the domain of the given function.

$$f(x) = \frac{\sqrt{x+7}}{x-4}$$

=> We have 2 things to consider here.

What makes the denominator zero? $x = 4$

What makes the radicand (the expression under the radical symbol) negative?

$x + 7 < 0 \Rightarrow x < -7$

These *x*s make the function undefined. So, the domain is $[-7,4) \cup (4,\infty)$

So the **correct answer is A**.

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3) Let $f(x) = 2x^2 + 3x$. Evaluate the difference quotient $\frac{f(x+h)-f(x)}{h}$.

J $$\frac{f(x+h)-f(x)}{h} = \frac{2(x+h)^2 + 3(x+h) - [2x^2+3x]}{h} = \frac{2(x^2+2hx+h^2)+3x+3h-2x^2-3x}{h}$$

$$= \frac{2x^2+4hx+2h^2+3h-2x^2}{h} = \frac{h(4x+2h+3)}{h} = 4x+2h+3$$

Correct answer is C.

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4) Which of the following represents the graph of a quadratic function satisfying the following conditions: $b^2 - 4ac < 0$ and $a > 0$.

I won't put all the choices here. To answer this question you want to think of the quadratic formula.

$b^2 - 4ac < 0$ means that the **radicand** in the quadratic formula is **negative** and so there are no zeros of the quadratic expression. So – the graph must lie either totally above the *x*-axis or totally below the *x*-axis.

$a > 0$ means the parabola 'opens up' – is concave up – and has a minimum.

So the graph must be totally above the *x*-axis and opening up. Since it is a **quadratic** (degree=2) there is only one turning point.

There is only one graph that matches all these criteria (see your key …..).

---

5) Determine whether the graph of $f(x)=\dfrac{x^4}{x^2-1}$ has:

=> To determine which kinds of symmetries might apply, we note that $f(a)=\dfrac{a^4}{a^2-1}$.

A) x-axis symmetry.

What we need to check is that if $k=f(x)$ then also $-k=f(x)$ for the same x … for every x.

This can only happen if $f(x)=0$ for every x or $f(x)$ is not a function. All rational expressions like the one we have **are** functions, so this is false.

B) y-axis symmetry.

We need to check that $f(-a)=f(a)$.

$$f(-a)=\frac{(-a)^4}{(-a)^2-1}=\frac{a^4}{a^2-1}=f(a)$$

So we have y-axis symmetry.

C) Origin symmetry.

We need to check that $f(-a)=-f(a)$.

Well – we just showed that $f(-a)=f(a)$ so this is false.

D) Both origin and y-axis symmetry.

Not true – because of (C).

E) None of the preceding. Also false.

Correct answer is B.

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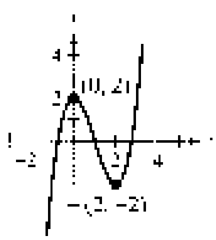
6) The graph of a function f is shown below. On what interval(s) is $f(x)\ge 0$?



Those xs are the ones where the graph is **above** (or on) the x-axis. So - $[-0.75,1]\cup[2.75,\infty)$.

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7. A farmer with 400 meters of fencing wants to enclose a rectangular plot that borders on a river. If the farmer does not fence the side along the river, what is the largest area that can be enclosed?

Draw a diagram, and let x be the side opposite the river. Then y are the other two sides of the fence. So $x+2y=400 \Rightarrow x=400-2y$. The area enclosed is $A=xy \Rightarrow A=(400-2y)y$. This quadratic is concave down, and has a maximum on the line of symmetry. The line of symmetry is midway between the two zeros. The zeros are $400-2y \Longrightarrow y=200$ and $y=0$. So $y=100$ represents the maximum of the area. The dimensions are 100 x 200. **The largest area is 20,000 square meters.**

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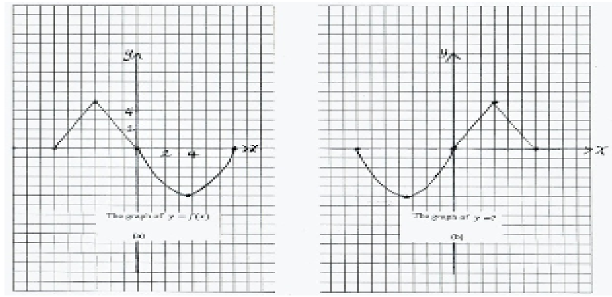
8) Figure (I) is the graph of $y=f(x)$. What was done to graph (I) to obtain the graph in Figure (II).



The answer is $y = -f(x)$. It's rotated around the y-axis.

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9) Graph $f(x) = 2x^2 - 6x + 18$. Which of the following statements is true?

A) The largest x-intercept is $\dfrac{3+3\sqrt{3}}{2}$.

The x-intercepts are the zeros – so we can use the quadratic formula on $f(x) = 2\left(x^2 - 3x + 9\right)$.

We have $x = \dfrac{-(-3) \pm \sqrt{(-3)^2 - 4(1)(9)}}{2(1)} = \dfrac{(3) \pm \sqrt{9-36}}{2(1)}$. This tells us there are **NO ZEROS**. So (A) is false.

B) The function f is increasing on the interval $\left(-\infty, \dfrac{3}{2}\right)$. Our parabola is concave up with a minimum, so the increasing part is on the right half (a, ∞). So this is false.

C) The axis of symmetry is $y = \dfrac{27}{2}$. Remember:

* the axis of symmetry is midway between the zeros, and
* the axis of symmetry doesn't change as I move the parabola up or down the y-axis. So I look at the related function $f(x) = 2\left(x^2 - 3x\right) = 2x(x-3)$. The zeros are 0 and 3, so the axis of symmetry is $x = \dfrac{3}{2}$. C) is false.

D) The maximum value of the function is 27/2. As we discuss already, this function does not hace a maximum value. False.

E) None of the statements are true. **This is the correct one**.

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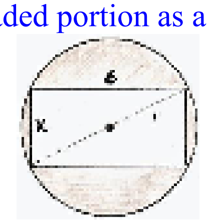
10) A rectangle whose length is 6 and width is x is inscribed in a circle of radius r. Use the following figure to express the area of the shaded portion as a function of r.



You'll notice from the picture (Pictures are SO helpful!) that we have a triangle involving x, 6, and the diameter (which is $2r$). So $6^2 + x^2 = (2r)^2 \Rightarrow x^2 = 4r^2 - 36 \Rightarrow x = \sqrt{4r^2 - 36} = \sqrt{4}\sqrt{x^2 - 9} = 2\sqrt{x^2 - 9}$.

The area of the shaded region is the area of the entire circle (πr^2) minus the area of the rectangle (which is $6x = 6\left(2\sqrt{r^2 - 9}\right) = 12\sqrt{r^2 - 9}$). So the **correct answer is (B).**

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11) $h(x)=\left(\frac{x+4}{x+1}\right)^2$. Find two functions *f* and *g* so that $h(x)=(f\circ g)(x)$.

A) $f(x)=\frac{x^2+16}{x^2+1}; g(x)=x$. When we compose the two functions, we replace every "*x*" in the rule for *f*(*x*) with the rule for *g*(*x*) … which is just *x*! So $(f\circ g)(x)=\frac{x^2+16}{x^2+1}$.

B) $f(x)=x; g(x)=\frac{x^2+16}{x^2+1}$. *f*(*x*) says 'left the input in parentheses alone.' So the result is the same as (A).

C) $f(x)=\frac{x+4}{x+1}; g(x)=x^2$. Replace every *x* in *f* with x^2: $(f\circ g)(x)=\frac{(x^2)+4}{(x^2)+1}$

D) $f(x)=x^2; g(x)=\frac{x+4}{x+1}$. Replace the *x* in *f* with the rule for *g*(*x*): $(f\circ g)(x)=\left(\frac{x+4}{x+1}\right)^2$. And **this is the correct answer!**

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12) Let $f(x)=x^2+3$ and $g(x)=\sqrt{x-8}$. What is the domain of the composition $(f\circ g)(x)$?

The point of this question is to see if you know to look at the domain of g(x) instead of the final result. **The domain of the final composition can not add *x* values to the domain of *g*.**

So first we examine the domain of g. It's a square root, so we look at the radicand (since we are taking an **even** root the domain will be whatever *x*s make the radicand zero or positive.

$x-8\geq 0 \Rightarrow x\geq 8$.

Now – the function *f* **can take away** *x*s in the domain of *g* … but that doesn't happen here. So the **correct answer is** $[8,\infty)$.

---

13) Which of the following functions is one-to-one?

A function is one-to-one if it passes the horizontal line test as well as the vertical line test. The importance of this is that THEN we know that the function has an inverse (that is ALSO a function).

I. $f(x)=(x-1)^3+2$. We know that the basic cubic function $y=x^3$ is one-to-one. That is **not** true of all cubics, but it is true of $y=x^3$. So what do we do to that basic cubic? We've replaced *x* with (*x* – 1) – recall that this is a simple horizontal translation (movement) of one unit to the right. That sure won't change the shape, so it is still one-to-one. The other change we introduce is a vertical translation of 2 units up. That also doesn't change the shape. So **I is one-to-one**.

II. $f(x)=(x-1)^2+2$. We **know** that **NO** quadratic is one-to-one without restricting the domain. So this was the sucker choice.

III. $f(x)=x-1$. All non-horizontal lines are one-to-one (well, they can't be vertical lines either if we're talking about functions …). So this one is as well.

The **correct answer, therefore, is I and III.**

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14) If $f(x)=(x+5)^2$ for $x\geq -5$, what is the domain of $f^{-1}(x)$?

We ask ourselves what the range is of our function. Notice that x = -5 is the only zero of our function, and the parabola is concave up, so the range is $[0,\infty)$. When we take the inverse of a function we are switching the domain and range. So the domain of $f^{-1}(x)$ will be $[0,\infty)$.

---

15) Which of the following is/are true?

I. To graph the function $g(x) = f(x+3)$, shift the graph of *f* to the right three units.

No … think about it this way … How do we get back to the origin on the original graph of *f*? Well – we get there from a value of *x* that makes $(x + 3) = 0$. That value is **negative** 3. So it's a shift to the LEFT three units.

II. $[1.75] = 2$ This is the greatest integer function – one of the 'important functions' on that one page we mentioned in class. There were several homework problems or examples using it. This statement is **FALSE**. The greatest integer function of 1.75 is 1.

III. All polynomial functions of even degree must have at least one turning point. This is **TRUE**! Remember that the end behavior of an even degree polynomial is the same in both directions – either both sides going to +infinity or both sides going to –infinity. So there must be at least one turning point.

Correct answer: C on the exam I'm looking at anyway ☺

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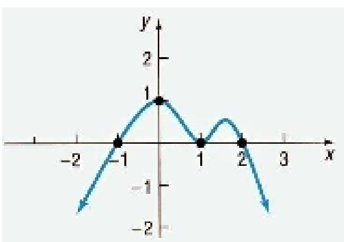
16) The graph of a function *f*, is given below:



Which of the following functions might have this graph?

=>

What we notice first are the zeros: $x = -1, 1, 2$. The zeros **tell us what the factors are**.

Next we look at the 'multiplicities' of the roots. Essentially if the graph of the function goes **through** the zero, that factor (root) will have an odd multiplicity (in other words an odd exponent of the factor). If the graph of the function touches the zero and **bounces back**, then the multiplicity will be even.

Third, we look at the *y*-intercept to get an idea of the constant that might multiply the whole product.

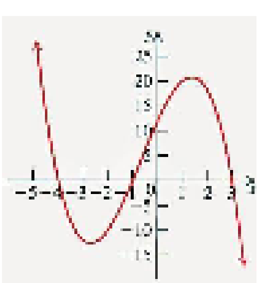
So we are looking for a function like: $f(x) = (x-(-1))(x-1)(x-2) = (x+1)(x-1)(x-2)$

However, the multiplicity of the root +1 is even, so we modify that slightly: $(x+1)(x-1)^2(x-2)$.

But wait! If we multiply out the constant terms, we see we have a *y*-intercept of -2. We want (from the graph) a *y*-intercept of +1. So we have to multiply by -1/2.

Our final try: $-\frac{1}{2}(x+1)(x-1)^2(x-2)$

And that was **the correct answer**.



17) A graph of a function is shown below. How many inflection points does the function have?

An inflection point is a point where the concavity changes. Since the 'left half' is concave up, and the right half is concave down, **there is one inflection point!**